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THE LINEAR PROBLEM OF HYDROFOIL MOVING UNDER AN INTERFACE BETWEEN TWO HEAVY FLUIDS

D. N. Gorelov and S. I. Gorlov

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The problem of a hydrofoil moving under the interface between two media has found wide practical applications and is a subject of interest to many researchers. A systematic description of the results obtained is given in [1]. Most papers consider a linear problem of hydrofoil motion under a free surface. The basic results were obtained by M. V. Keldysh, M. A. Lavrent'ev [2], and N. E. Kotchin [3]. They derived exact solutions of the problem of vortex motion under the free surface of a heavy liquid that allowed the boundary-value problem of hydrofoil motion to be reduced to integral equations. Later studies were mainly devoted to the methods of solving the corresponding integral equations.

Much less is known about the general case where the second medium is not the vacuum. The results obtained are reviewed by Sturova [4].

We consider the problem of hydrofoil motion under the interface between two media in a more general statement. The linear boundary-value problem is reduced to two integral equations. Their kernels are the exact solution of the vortex problem. We developed an effective algorithm for solving these two equations which is applicable to hydrofoils of any thickness, including an arbitrary small one. For the Joukowski profile, the calculation results are given for lift, wave resistance, the moment and shape of the interface between media, depending on the problem parameters.

1. Let us consider a linear boundary-value problem, which describes the motion of hydrofoil L under the interface between two liquid media D_1 and D_2. We introduce a hydrofoil-related coordinate system Oxy by orienting the Ox axis along the unperturbed interface (Fig. 1). We assume that the liquid in layers D_1 and D_2 is ideal, incompressible, heavy, and homogeneous, and the motion of liquid beyond the interface and the contour L is stationary and potential. The notation is as follows: g is the acceleration of gravity; H is the distance between the hydrofoil leading edge and the unperturbed-medium interface; b is the hydrofoil chord; α is the angle of attack; ρ_k is the liquid density in the kth layer; and $V_{k\infty}$ is the liquid velocity at infinity in front of the hydrofoil in the D_k layer ($k = 1$ and 2).

The liquid motion in each D_k layer is described by the complex velocity $\overline{V}_k(z)$, $z = x + iy$. We require that the functions $\overline{V}_k(z)$ be analytical in D_k (beyond L for $k = 1$) and satisfy the following boundary conditions: the continuity of both pressure and the normal velocity component upon passage through the interface between two media, the decay of velocity perturbations at infinity in front of the hydrofoil in the D_1 and D_2 regions, the absence of liquid flow through contour L, and the Joukowski postulate on the trailing edge.

The functions below satisfy the above conditions, except for the last two:

$$\overline{V}_k(z) = V_{k\infty} + \frac{1}{2\pi i}\int\limits_L K_k(z,\zeta)\,\gamma(s)e^{-i\theta(s)}d\zeta, \quad k = 1, 2. \tag{1.1}$$

Here s is the arc coordinate of point $\zeta \in L$; $\gamma(s)$ is the intensity of the vortex layer simulating L; $\theta(s)$ is the angle between the tangent to L at the point $\zeta(s)$ and the Ox axis;

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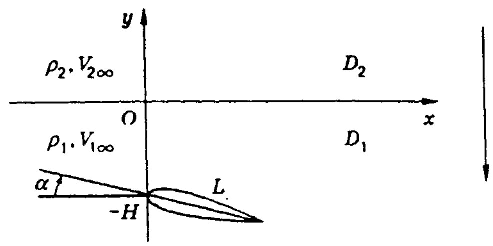

Fig. 1

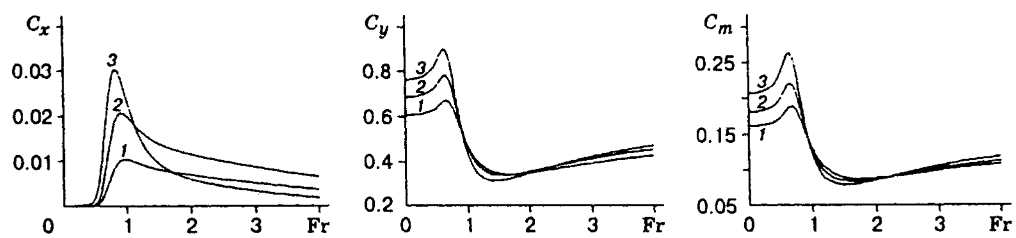

Fig. 2

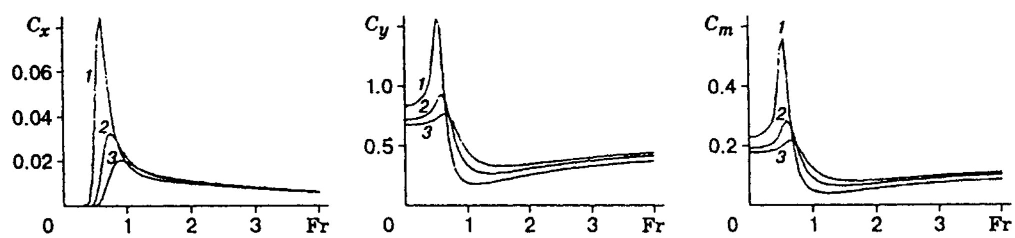

Fig. 3

$$K_1(z,\zeta) = \frac{1}{2\pi i}\,\frac{1}{z-\zeta} + \frac{m_{12}}{2\pi i}\,\frac{1}{z-\bar\zeta} + \frac{\nu_1 m_{12}^1}{\pi}\int\limits_0^\infty \frac{e^{-i\lambda(z-\bar\zeta)}}{\lambda-\nu_1}\,d\lambda - \nu_1 m_{12}^1 i e^{-i\nu_1(z-\bar\zeta)}; \qquad (1.2)$$

$$K_2(z,\zeta) = \frac{V_{2\infty}}{V_{1\infty}}\left\{\frac{m_{12}^1}{\pi i}\,\frac{1}{z-\zeta} - \frac{\nu_1 m_{12}^1}{\pi}\int\limits_0^\infty \frac{e^{i\lambda(z-\zeta)}}{\lambda-\nu_1}\,d\lambda - \nu_1 m_{12}^1 i e^{i\nu_1(z-\zeta)}\right\}, \qquad (1.3)$$

where

$$m_{12}^1 = \frac{\rho_1 V_{1\infty}^2}{\rho_1 V_{1\infty}^2 + \rho_2 V_{2\infty}^2};\quad \nu_1 = \frac{g(\rho_1-\rho_2)}{\rho_1 V_{1\infty}^2 + \rho_2 V_{2\infty}^2};\quad m_{12} = m_{12}^1 - m_{12}^2;\quad m_{12}^2 = \frac{\rho_2 V_{2\infty}^2}{\rho_1 V_{1\infty}^2 + \rho_2 V_{2\infty}^2}.$$

Expressions (1.2) and (1.3) for $K_k(z,\zeta)$ ($k = 1$ and 2) are exact solutions of the corresponding boundary-value problem of a vortex of unit intensity [5] which proved to be more convenient than the solution of N. E. Kotchin [3].

The condition for a smooth steady flow about the contour L can be written as

$$\operatorname{Im}\{\overline{V}_0(z)e^{i\theta(s)}\} = 0, \quad z \in L; \qquad (1.4)$$

$$-\frac{1}{2}\gamma(s) = \operatorname{Re}\{\overline{V}_0(z)e^{i\theta(s)}\}, \quad z \in L, \qquad (1.5)$$

where $\overline{V}_0(z) = \overline{V}_1(z)$ with $z \in L$. In this case, a special integral in (1.1) is used as the Cauchy principal value.

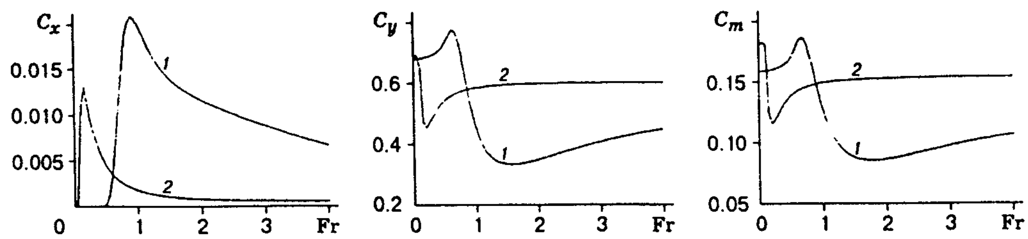

Fig. 4

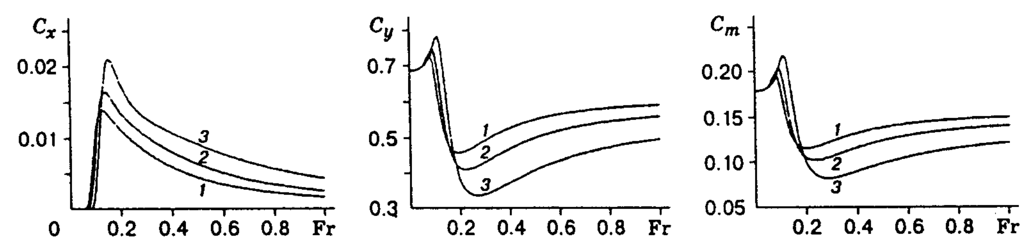

Fig. 5

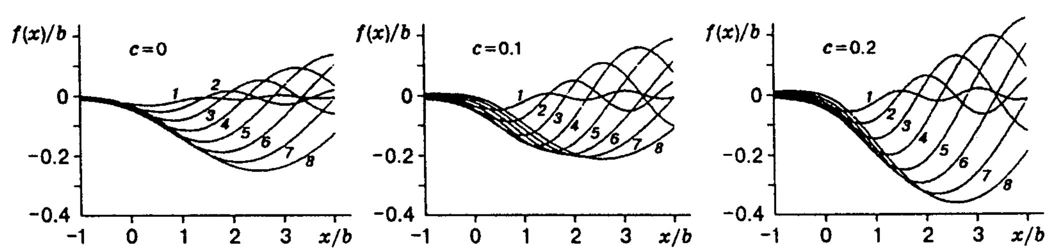

Fig. 6

According to [6], the independent Eqs. (1.4) and (1.5) reduce to a system of two integral equations that do not degenerate in the limiting case of an infinitely small hydrofoil thickness. A method of solving this system in the class of functions $\gamma(s)$ which satisfy the Joukowski postulate was proposed in [7]. The form of the interface between two media can be found by the equation $y = f(x)$, where

$$f(x) = -\frac{1}{\nu_1}\operatorname{Re}\Big\{m_{12}^1\Big(\frac{\overline{V}_1(z)}{V_{1\infty}} - 1\Big) - m_{12}^2\Big(\frac{\overline{V}_2(z)}{V_{2\infty}} - 1\Big)\Big\}, \quad z = x.$$

The pressure distribution over the hydrofoil, the total hydrodynamic forces R_x and R_y, and the moment M were calculated as described in [8].

2. Calculations were performed for a symmetric Joukowski profile. The calculation algorithm was tested, using the well-known solutions of the problem of an infinite liquid flow about the Joukowski profile and the motion of a plate under the free surface of a heavy liquid [9]. In this case, the relative calculation error was not more than 1%.

The dimensionless parameters of the problem are the Froude number $\mathrm{Fr} = V_{1\infty}/\sqrt{gb}$; the ratio of the densities $\rho_* = \rho_2/\rho_1$; the ratio of flow velocities $v_* = V_{2\infty}/V_{1\infty}$; the distance between the leading edge and the unperturbed interface $h = H/b$.

A numerical experiment was performed to estimate the effect of these parameters on the wave resistance, lift, and moment relative to the leading edge moving under the interface between two heavy liquids. The main results are shown in Figs. 2–6.

Figure 2 shows the standard coefficients C_x, C_y, and C_m versus the Froude number for the relative thickness $c = 0, 0.1$, and 0.2 (curves 1–3) with $\alpha = 5^0$, $h = 1$, and $\rho_* = 0$. Near Fr = 1 a substantial increase in the wave resistance, lift, and moment is observed with increase in the hydrofoil thickness at a fixed distance h. A similar effect occurs with decreasing distance between the hydrofoil and the interface for a fixed thickness c (Fig. 3, where $c = 0.1$, $\alpha = 5^0$, and $\rho_* = 0$; curves 1–3 correspond to $h = 0.5, 0.75$, and 1). It is interesting to compare the curves of C_x, C_y, and C_m versus the Froude number at $\rho_* = 0$ (a free surface) and $\rho_* = 0.97$ (the salt-sweet water interface). The calculation results for $c = 0.1$, $\alpha = 5^0$, $h = 1$, and $v_* = 1$ are shown in Fig. 4 (curves 1 and 2 correspond to $\rho_* = 0$ and 0.97). The effect of the ratio of the flow velocities on the dependences of C_x, C_y, and C_m on Fr for $c = 0.1$, $\alpha = 5^0$, $h = 1$, $\rho_* = 0.97$; and $v_* = 1.0, 0.5$, and 0 (curves 1–3) is schematically depicted in Fig. 5.

The influence of the Froude number on the interface shape is illustrated in Fig. 6 for a free surface: Fr = 0.5, 0.6, 0.7, 0.8, 0.9, 1.0, 1.1, and 1.2 (curves 1–8) for $c = 0, 0.1$, and 0.2; $\alpha = 5^0$; $h = 1$.

The numerical results allow us to draw the following conclusions on the effect of the problem parameters on the wave resistance and hydrofoil lift. The main characteristic is the dependence of C_x, C_y, and C_m on the Froude number. The influence of the other parameters is most significant for Fr ~ 1 and Fr < 1. In this case, an increase in the relative hydrofoil thickness has the same effect as the approach of the hydrofoil to the interface. An increase in the ratio of densities ρ_* from 0 to 1 leads to both a decrease in the wave resistance for all Fr values and the shift of maxC_x to the left, whereas the dependence of the lift and moment on the Froude number at different ρ_* appears to be more complex. The dependence of C_x, C_y, and C_m on the parameter v_* is noticeable only for Fr < 0.5.

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